Thermal Physics

Exam T2

Wednesday, November 9 2022, 8:30-10:30, Aletta Jacobshal

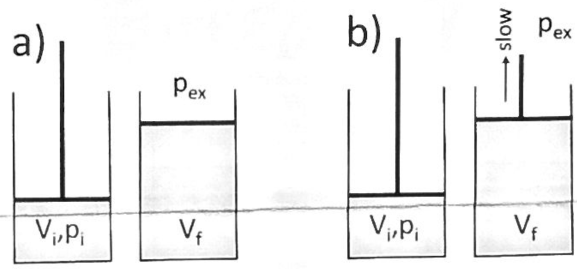
The total number of points that can be reached in this exam is 90. The exam consists of **4** problems.

Final grade = (points/10) + 1.

Note: For efficient (=quick) grading, we need to have clearly separated answers. Please start a new page when you start answering a new problem.

1) Expansion work and reversibility (30 pt)

a) A cylinder is filled with 1 mole of an ideal gas ($T = 25$ C, $p = 14$ bar). Initially, the gas is compressed by a massless piston that can move without friction once it is released from its initial position. The ideal gas will subsequently be expanded to an external pressure of 1 bar and to a volume of 23.5 liter. The piston is unlocked and the gas is allowed to expand freely until it reaches the external pressure. Calculate the expansion work **(8 pt)** .



b) Start from the same initial situation as in a). Now the piston is unlocked and the gas is only allowed to expand very slowly until it reaches the external pressure of p=1 bar. The process is isothermal. Calculate the expansion work. **(8 pt)**.

c) Draw a p-V diagram for both processes and indicate the expansion work. **(8 pt)**.

d) Which process (a or b) can be considered thermodynamically reversible and why? **(6 pt)**.

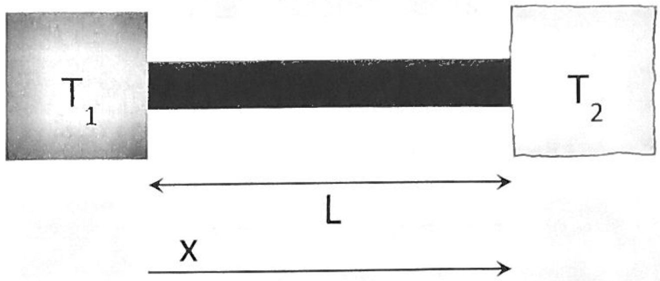
2) Thermal diffusion (25 pt)

Consider the thermal diffusion equation in 1 dimension:

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$$

A straight iron wire of length L connects two large metal blocks at temperatures T_1 and T_2 (see sketch).

a) Derive a functional expression for the wire temperature as a function of x in the steady state **(10 pt)**.



b) If the wire has a cross sectional area of 1 mm², a length $L = 0.1$ m and a thermal conductivity of 79.5 Wm⁻¹K⁻¹, how much thermal energy is transported through the wire in 100 seconds ($T_1 = 200$ K, $T_2 = 300$ K) **(15 pt)**?

3) Heat engine (30 pt)

A heat engine is operating between two heat reservoirs 1 and 2, that have constant temperatures of $T_1 =$ 450 K and T_2 (with T_2 unknown). The entire system (the two reservoirs and the engine) is thermally isolated. The efficiency of the engine is 20% ($\eta = 0.2$). During the cycle, 80 J of work is delivered by the engine. The increase in total entropy after one cycle is $\Delta S_{tot} = 0.25$ J/K.

a) Assume $T_2 < T_1$. Show with a calculation why $q_1 = 400$ J, with q_1 the amount of heat extracted from reservoir 1, and why $q_2 = 320$ J, with q_2 the amount of heat deposited in reservoir 2. (10 pt)

b) Assume $T_2 > T_1$. Calculate q_1 and q_2 for this situation. Explain explicitly whether heat is being deposited or extracted into or from each of the reservoirs. (10 pt)

c) Calculate the temperature T_2 (hint: use the total entropy). Assume that both heat reservoirs have very large, constant volume. Provide solutions for the cases $T_2 < T_1$ and $T_2 > T_1$. (10 pt)

4) Maxwell relation (5 pt)

Derive the Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$.

Constants:

Avogardro's number:	$N_A \approx 6.02 \times 10^{23}$ mol⁻¹
Boltzmann constant:	$k_B \approx 1.381 \times 10^{-23}$ J/K
Gas constant:	$R = 8.31$ JK⁻¹mol⁻¹
Atomic mass unit (u):	$m_u \approx 1.67 \times 10^{-27}$kg
Electronvolt:	1 eV $\approx 1.6 \times 10^{-19}$ J

$k_B := 1.380649 \cdot 10^{-23}$ J/K exactly